Lec 3: 08/31/2010

Solar Luminosity and Composition:

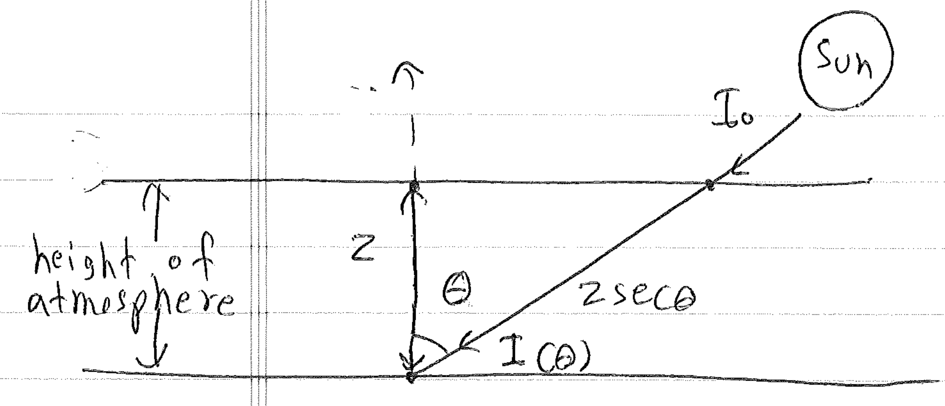
Most of visible light comes from photosphere of Sun. The mean free path of photons becomes long enough in the photosphere such that they can leave Sun without further scattering. The light from photosphere has two components:

1) Continuum.

2) Absorption lines (Fraunhofer lines).

One can use Continuum to find solar luminosity, while absorption lines can be used to find composition of the photosphere.

First lets consider the Continuum. Total radiation arriving on the Earth can be calculated by measuring $I(\theta)$ (I being the intensity measured on Earth's surface and θ being the viewing angle):

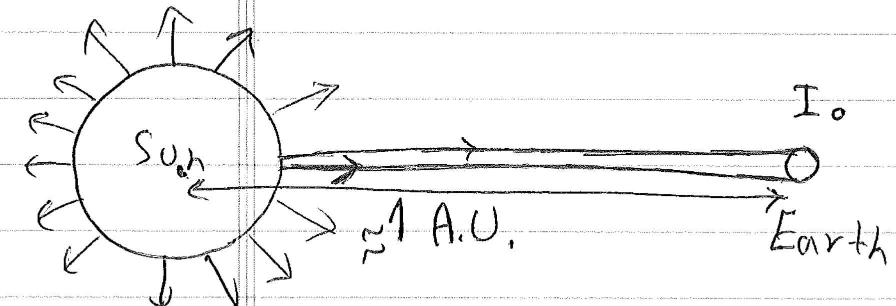


$$I_{(\theta)} = I_o\, e^{-kz\sec\theta}$$

(k: atmosphere absorption coefficient)

I_o can be obtained by extrapolating $I_{(\theta)}$ to (unphysical) value of $\sec\theta = 0$. It turns out that 40% of I_o reaches Earth's surface in temperate zone. The rest is scattered or absorbed by atomsphere. The atmosphere is virtually opaque for $\lambda < 2900$ Å because of Ozone (O_3).

Once we have I_o, we can find solar luminosity $L_\odot$:



$$\boxed{L_\odot = 4\pi (1\,A.U.)^2\, I_o = (3.90 \pm 0.04) \times 10^{33}\ erg/s}$$

Assuming Sun is a black body, we can find photosphere's temperature :

$$L_\odot = 4\pi R_\odot^2 \sigma T_e^4$$

(σ: Stefan–Boltzmann Constant $= 5.66 \times 10^{-5} \frac{erg}{cm^2 K^4 s}$)

$$\Rightarrow \boxed{T_e \approx 5760 \text{ K}}$$

Actual spectrum of the photosphere **is a fair approximation** to that of a black body, but it is deficient in the ultraviolet.

To put things in perspective, let us estimate Sun's lifetime from its luminosity. The main source of Sun energy is Hydrogen burning with the key reactions as follows:

$$\left\{\begin{array}{l} p + p \rightarrow {}^2D + e^+ + \nu \\ p + {}^2D \rightarrow {}^3He + \gamma \\ {}^3He + {}^3He \rightarrow {}^4He + 2p \end{array}\right. \Longrightarrow 4p + 2e^- \rightarrow {}^4He + 2\nu + 26.5 \text{ MeV}$$

The 26.5 MeV in energy is the difference between the masses of initial state ($4p$) and final state (${}^4He$):

$\Delta M = 4 \times 1.0081\ m_u - 4.0039\ m_u$ (m_u: atomic mass unit)

$= 931.49\ \frac{MeV}{c^2} = 1.6605 \times 10^{-27}$ kg

$\Delta M c^2 = 26.5$ MeV

The energy released in Hydrogen burning amounts to 0.7% of the mass involved.

Using the fact that Sun is in hydrostatic equilibrium, we can find mass loss rate from its luminosity:

$$\frac{L_\odot}{c^2} = 4.25 \times 10^{12}\ g\,s^{-1} = \boxed{3.75 \times 10^{38}\ p\,s^{-1}}$$

The total number of protons is 1.2×10^{57} p. The time needed for one solar mass of Hydrogen converted to Helium is then found to be 3×10^{18} s, which is $\sim 10^{11}$ years.

$\therefore$ This is clearly larger that the present age of Sun ($\sim 5 \times 10^9$ y), and also much larger than the age of universe ($\sim 14 \times 10^9$ y).

A more precise estimate can be found as follows.

Theory of stellar evolution shows that Sun will remain relatively unchanged until ~10% of its Hydrogen is converted to **Helium**. The amount of time t that it can shine with luminosity $L_\odot$ is then found to be:

$$\boxed{t \simeq 3 \times 10^{17} \text{ sec} \simeq 10^{10} \text{ yr}}$$

Again, present age is comfortably within this limit.

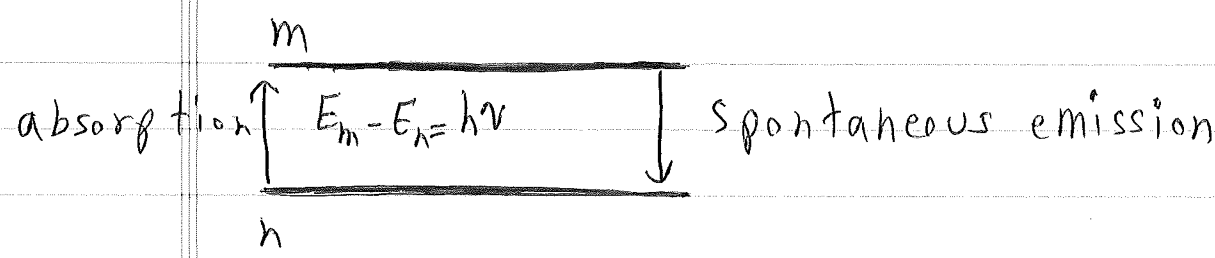
Next lets move to the absorption lines in the photosphere spectrum. There are 1000's of them; lines corresponding to every stable element in the periodic table has been discovered.

The question is how to deduce elemental abundance from the absorption lines. To do this we need three things:

1) To positively identify line as belonging to the given element.

2) Physical condition under which the line is excited (temperature, pressure, densities, surface gravity, etc).

3) Intrinsic strength of line, which comes from atomic physics. We need to know natural lifetime of excited atoms and ions (equivalently, Einstein "A" coefficient, $A = \frac{1}{\tau}$).



Energy emitted per unit time $= h\nu A_{mn}$

One can calculate A for all of Hydrogen transitions, and do approximate calculations for Hydrogenic systems. As an example, for $2p \rightarrow 1s$ (Lyman-α) transition in Hydrogen we have ($\lambda = 1216$ Å):

$$\therefore A = \frac{2^{17}}{3^{10}} \frac{e^2 \omega^3}{\hbar c^2} a_0^2 \quad \left(a_0 \equiv \frac{\hbar^2}{m_e e^2} \right) \Rightarrow \boxed{A = 6 \times 10^8 \ s^{-1}}$$

From measuring the absorption lines we can find Nh, where N is the number of absorbing atoms per cm^3 and h is the effective height of absorbing layer.

Knowledge of N_h only tells us abundance of a given state of the particular absorbing atom. We want to know abundance of all states (total abundance of atoms and ions). In order to do this we need to turn to Saha equation (derivation of it will be on one of the assignments).

According to the Saha ionization equation, for the process $A \rightleftarrows A^+ + e^-$ in thermal equilibrium we have:

$$\frac{N_{A^+} N_e}{N_A} = f(T) \qquad (T: \text{temperature})$$

By using Saha and Boltzmann equations we can find the total number of atoms and ions of the element considered, as well as N_e and T. This is clearly an iterative process; we note that N_e comes from all elements.

One may naively think that $N_e \simeq N_p$, and hence $P \simeq 2P_e$.

However, at $T \sim 6000$ K, H is not fully ionized. In fact, metals are the main contributors to N_e. From the calculations described above, we find:

$$\boxed{P_e \simeq 100 \, \frac{\text{dyne}}{\text{cm}^2} , \quad P \sim 10^4 \, P_e}$$

Where P comes from all particles, including electrons.